Integral of sec(x):

$$\int \frac{1}{\cos(x)}\text{dx} = \int \frac{\cos(x)}{\cos^2(x)}\text{dx} = \int \frac{\cos(x)}{1-\sin^2(x)}\text{dx}$$

Now use u - substitution and sub in u for sin(x). Then use partial fractions to get...

$$\int \frac{1}{1-u^2}\text{du} = \frac{1}{2}\int \left(\frac{1}{1-u} + \frac{1}{1+u}\right)\text{du} = \frac{1}{2}(\ln|u+1| - \ln|u-1|) + C = \frac{1}{2}\ln\left|\frac{u+1}{u-1}\right| + C$$

To get the final anser substitute sin(x) back in for u, and use some properties of logarithms to simplify.

$$\frac{1}{2}\ln\left|\frac{\sin(x)+1}{\sin(x)-1}\right| + C = \frac{1}{2}\ln\left|\frac{\sin(x)+1}{\sin(x)-1}\left(\frac{\sin(x)+1}{\sin(x)+1}\right)\right| + C = \frac{1}{2}\ln\left|\frac{(\sin(x)+1)^2}{1-\sin^2(x)}\right| + C$$

$$= \frac{1}{2}\ln\left|\left(\frac{(1+\sin(x))}{\cos(x)}\right)^2\right| + C = 2\,\frac{1}{2}\ln\left|\left(\frac{(1+\sin(x))}{\cos(x)}\right)\right| + C = \ln\left|\frac{1}{\cos(x)} + \frac{\sin(x)}{\cos(x)}\right| + C$$

$= \mathbf{\ln|\sec(x) + \tan(x)|} + C$

Alternative solution (somewhat easier):

$$\int \sec(x)\text{dx} = \int \left(\sec(x)\left(\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}\right)\right)\text{dx} = \int \left(\frac{\sec^2(x)+\tan(x)\sec(x)}{\sec(x)+\tan(x)}\right)\text{dx}$$

Use u substtition with u = sec(x) + tan(x) to get...

$$\int \frac{\text{du}}{u} = \ln|u| = \ln|\sec(x)+\tan(x)| + C$$

Notice this is the same equation obtained before.