Integral of csc(x):

$$\int \frac{1}{\sin(x)}\text{dx} = \int \frac{\sin(x)}{\sin^2(x)}\text{dx} = \int \frac{\sin(x)}{1-\cos^2(x)}\text{dx}$$

Now use u - substitution and sub in u for cos(x). Then use partial fractions to get...

$$\int \frac{1}{1-u^2}\text{du} = -\frac{1}{2}\int \left(\frac{1}{1-u} + \frac{1}{1+u}\right)\text{du} = -\frac{1}{2}(\ln|u+1| - \ln|u-1|) + C = \frac{1}{2}\ln\left|\frac{u-1}{u+1}\right| + C$$

To get the final anser substitute cos(x) back in for u, and use some properties of logarithms to simplify.

$$\frac{1}{2}\ln\left|\frac{\cos(x)-1}{\cos(x)+1}\right| + C = \frac{1}{2}\ln\left|\frac{\cos(x)-1}{\cos(x)+1}\left(\frac{\cos(x)-1}{\cos(x)-1}\right)\right| + C = \frac{1}{2}\ln\left|\frac{(\cos(x)-1)^2}{1-\cos^2(x)}\right| + C$$

$$= \frac{1}{2}\ln\left|\left(\frac{(1-\cos(x))}{\sin(x)}\right)^2\right| + C = 2\,\frac{1}{2}\ln\left|\left(\frac{(1-\cos(x))}{\sin(x)}\right)\right| + C = \ln\left|\frac{1}{\sin(x)} - \frac{\cos(x)}{\sin(x)}\right| + C$$

$= \mathbf{\ln|\csc(x) - \cot(x)|} + C$

Alternative solution (somewhat easier):

$$\int \csc(x)\text{dx} = \int \left(\csc(x)\left(\frac{\csc(x)-\cot(x)}{\csc(x)-\cot(x)}\right)\right)\text{dx} = \int \left(\frac{\csc^2(x) - \cot(x)\csc(x)}{\csc(x)-\cot(x)}\right)\text{dx}$$

Use u substtition with u = csc(x) - cot(x) to get...

$$\int \frac{\text{du}}{u} = \ln|u| = \ln|\csc(x) - \cot(x)| + C$$

Notice this is the same equation obtained before.